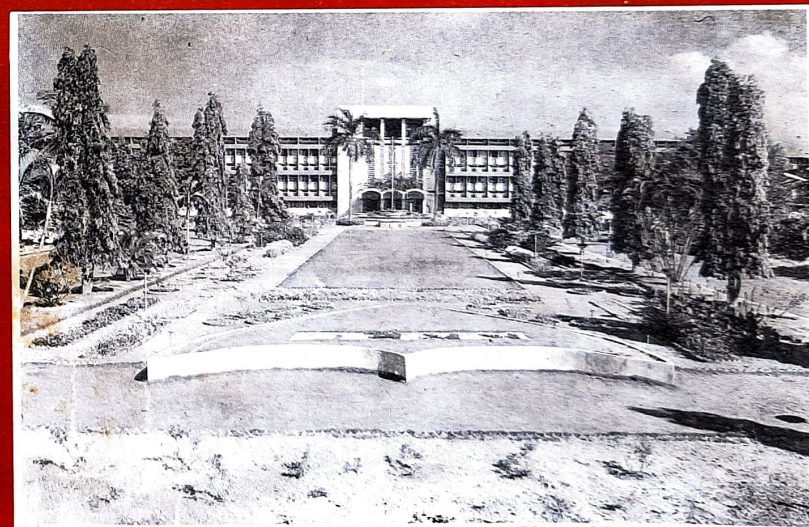


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# KREC RESEARCH BULLETIN

Vol. 9 Number 1

ISSN : 0971-3603

JUNE 2000

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## SYMMETRIC PRIMAL-DUAL SIMPLEX ALGORITHM FOR LINEAR PROGRAMMING

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### ABSTRACT:

*A novel iterative algorithm to solve linear programming (LP) problems based upon a newly identified set of four (and/or possibly six) distinct types of simplex pivoting steps defined over a symmetric primal-dual pair of LP, is presented. The two (or possibly four) types of classical (standard) simplex pivots are the Primal Standard Pivot with positive (and/or zero) indicator, and the Dual Standard Pivot with negative (and/or zero) indicator. The newly identified pivot types are: the Primal Tricky Pivot with positive indicator and the Dual Tricky Pivot with negative indicator.*

The Symmetric Primal-Dual Simplex Algorithm is an iterative solution strategy, wherein each iteration consists of the selection and application of exactly one pivoting step, chosen in a well defined systematic preference order (for our convenience, at least), from among the possibly four (or even six) distinct types of pivots, that may be valid depending on the tableau data entries.

For expressional convenience, this algorithm is discussed using the Tucker's Compact Simplex Tableau for Linear Programming problems expressed in the standard (canonical) form representing the Symmetric Primal-Dual Pair. An analysis of the evolution of the tableau data entries pattern observed as iterations proceed, is also presented. A classification of the LP problem into one of the six types is identified based on the data entries pattern observed in the final tableau.

### 1. INTRODUCTION:

Linear Programming (LP) problem represents one of the most widely used class of numerical/quantitative computational models, for which any possible improved solution technique would certainly be highly desirable. Of course, there has been several alternative solution strategies suggested including the classical simplex method of Dantzig [1] and several variations thereof, followed by recent polynomial time algorithms, namely the Ellipsoid Method of Khachiyan [2, 3] and the Karmarkar Algorithm [4] both classified now as belonging to Interior Point Algorithms.

In this paper, a novel generalization of the classical Simplex Method of Dantzig is presented. The fundamental concept in Dantzig's Simplex Method consists of moving from one basis (tableau) to a neighbouring basis (tableau), by a single exchange between a non-basic (entering) variable and a basic (leaving) variable, appropriately selected. We associate the

term "Simplex Algorithm" for our method because this fundamental feature is being preserved. The proposed algorithm is an enhancement over the Dantzig's Simplex Method, in terms of providing a wider scope for the selection of the pivots, as can be seen in the ensuing discussions. The development of the algorithm is mainly supported by the identification of a set of possibly four (or even six) distinct types of simplex pivot selections, maintaining a sense of symmetry between the primal and the dual problems. Also, for expressional efficiency or at least convenience, the solution strategy and the analysis thereof are all based on the Tucker's Compact Simplex Tableau for LP expressed in the standard (canonical) form representing the symmetric primal-dual pair. Hence we justify the chosen name for the present algorithm: "Symmetric Primal-Dual Simplex Algorithm for Linear Programming".

### 2. PRELIMINARIES:

We will go through some well known preliminaries for the sake of establishing the notational conventions used in this paper. The symmetric primal-dual pair of LP in the standard form is as follows.

Primal Problem:

$$\begin{array}{ll} \text{Maximize} & c.x = f \\ \text{s.t.} & A.x \leq b \\ & x \geq 0 \end{array} \quad (1)$$

Dual Problem:

$$\begin{array}{ll} \text{Minimize} & v.b = g \\ \text{s.t.} & v.A \geq c \\ & v \geq 0 \end{array} \quad (2)$$

The matrix-dimensional descriptions for each of the problem parameters/data in (1) and (2) above are as follows:

|   |                                  |            |
|---|----------------------------------|------------|
| x | Primal decision Variables        | nx1 vector |
| c | Primal objective function coeffs | 1xn vector |
| f | Primal objective function value  | 1x1 scalar |
| A | Primal constraint coeffs matrix  | mxn matrix |
| b | Primal constraint upper bound    | mx1 vector |
| v | Dual decision variables          | 1xm vector |
| g | Dual objective function value    | 1x1 scalar |

We introduce the mx1 vector y of slack variables to (1) and the 1xn vector u of surplus variables to (2) to write the symmetric primal-dual pair in the canonical form as follows

Primal Problem:

$$\begin{array}{ll} \text{Maximize} & c.x + 0.y = f \\ \text{s.t.} & A.x + I_m.y = b \\ & x, y \geq 0 \end{array} \quad (3)$$

Dual Problem:

$$\begin{array}{ll} \text{Minimize} & v.b + u.0 = g \\ \text{s.t.} & v.A - u.I_n = c \\ & v, u \geq 0 \end{array} \quad (4)$$

This primal-dual pair is represented in Tucker's Compact Simplex Tableau as:

$$\begin{array}{c}
 \begin{array}{cc|c}
 & x_j & -1 \\
 v_i & a_{ij} & b_i \\
 -1 & c_j & 0
 \end{array}
 \end{array}
 \begin{array}{l}
 = -y_i \\
 \\
 = f
 \end{array}
 \quad (5)$$

$\parallel$   $\parallel$   
 $u_j$   $g$

For the LP problem pair (1) & (2) or equivalently (3) & (4) the tableau (5) represents the initial tableau indicating the initial basic solution (IBS) wherein  $y_i$  are the primal basic variables in the initial basis associated (one to one permanent association) with  $v_i$  the dual non-basic variables, and  $x_j$  are the primal non-basic variables associated (one to one permanent association) with  $u_j$  the dual basic variables, in the initial basis. Note that  $x_j$  (and a -1) are column-labels and  $v_i$  (and a -1) are row labels in the tableau (5), and the way to interpret (read) the tableau is as follows

Primal Problem:

$$\sum_{j \in C} a_{ij} \cdot x_j - b_i = -y_i, \quad i \in R \text{ (row index)} \quad (6)$$

$$\sum_{j \in C} c_j \cdot x_j - 0 = f \text{ (to be maximized)}$$

Dual Problem:

$$\sum_{i \in R} v_i \cdot a_{ij} - c_j = u_j, \quad j \in C \text{ (Column index)} \quad (7)$$

$$\sum_{i \in R} v_i \cdot b_i - 0 = g \text{ (to be minimized)}$$

wherein the variables  $x_j$ ,  $y_i$ ,  $v_i$ ,  $u_j$  are all considered to be non-negative

### 3. ALGEBRA (ARITHMETIC) OF THE PIVOTING PROCESS:

It is an interesting but simple enough exercise to observe that once a pivot element is selected, the actual pivoting process (the algebra and hence the arithmetic operations) is the same irrespective of the pivot selection; for example whether it is a primal pivot or a dual pivot. Hence it suffices to present here a single (common) set of operations representing that pivoting process.

For the sake of generality, let us imagine that we are somewhere in the middle of solving a LP problem, and have the system model represented by a tableau as follows:

$$\begin{array}{c}
 \begin{array}{cc|c}
 & z_j^N & -1 \\
 w_i^N & \alpha_{ij} & \beta_i \\
 -1 & \gamma_j & \delta
 \end{array}
 \end{array}
 \begin{array}{l}
 = -z_i^B \\
 \\
 = f
 \end{array}
 \quad (8)$$

$\parallel$   $\parallel$   
 $w_j^B$   $g$

By the nature of the sequence of elementary row (column) operations being performed during any pivoting process, the system model represented by (8) is equivalent to that represented by the initial tableau (5) which corresponds to the primal-dual pair (6) & (7). The transformed version of the primal-dual pair directly expressed by (8) is as follows

Primal:

$$z_i^B = \beta_i - \sum_{j \in C} \alpha_{ij} \cdot z_j^N \quad i \in R \text{ (row index)}$$

$$f = -\delta + \sum_{j \in C} \gamma_j \cdot z_j^N \text{ (to be maximized)} \quad (9)$$

Dual:

$$w_j^B = -\gamma_j + \sum_{i \in R} w_i^N \cdot \alpha_{ij} \quad j \in C \text{ (column index)}$$

$$g = -\delta + \sum_{i \in R} w_i^N \cdot \beta_i \text{ (to be minimized)} \quad (10)$$

The effect of a pivoting operation on (9) & (10) performed with a chosen pivot element  $\alpha_{IJ}$  is exactly to affect an exchange between the variables indicated by I and J in (9) and (10). That is,  $z_j^N$  is entered into primal basis in exchange for  $z_i^B$  in (9), and  $w_i^N$  is entered into dual basis in exchange for  $w_j^B$  in (10). Suppose we have chosen the pivot element  $\alpha_{IJ}$  using some appropriate pivot selection scheme, and we would like to derive the resulting tableau. Let the resulting tableau be indicated as:



$$\begin{array}{cc|c}
 & (z_j^N)' & -1 \\
 (w_i^N)' & (\alpha_{ij})' & (\beta_i)' \\
 \hline
 -1 & (\gamma_j)' & (\delta)' \\
 & \parallel & \parallel \\
 & (w_j^B)' & (g)'
 \end{array} = - (z_i^B)' \quad (11)$$

Then the process of deriving (11) from (8) constitute the algebra (arithmetic) of the pivoting process and is detailed below (without any derivational justifications):

$$\begin{aligned}
 (\alpha_{ij})' &\leftarrow (1/\alpha_{ij}); & (\alpha_{ij})' &\leftarrow (\alpha_{ij})/\alpha_{ij}; & (\beta_i)' &\leftarrow (\beta_i/\alpha_{ij}); \\
 (\alpha_{ij})' &\leftarrow -(\alpha_{ij}/\alpha_{ij}); & (\gamma_j)' &\leftarrow -(\gamma_j/\alpha_{ij}); \\
 (\alpha_{ij})' &\leftarrow \alpha_{ij} - (\alpha_{ij}/\alpha_{ij}) \alpha_{ij}; & (\beta_i)' &\leftarrow \beta_i - (\beta_i/\alpha_{ij}) \alpha_{ij}; \\
 (\gamma_j)' &\leftarrow \gamma_j - (\alpha_{ij}/\alpha_{ij}) \gamma_j; & (\delta)' &\leftarrow \delta - (\beta_i/\alpha_{ij}) \gamma_j;
 \end{aligned}$$

followed by an exchange of the label associated with the row I and column J; that is effectively:

$$\begin{aligned}
 (z_i^N)' &\leftarrow z_i^B; & (z_i^B)' &\leftarrow z_i^N; \\
 (z_i^N)' &\leftarrow z_i^N; & (z_i^B)' &\leftarrow z_i^B;
 \end{aligned}$$

and

$$\begin{aligned}
 (w_i^B)' &\leftarrow w_i^N; & (w_i^N)' &\leftarrow w_i^B; \\
 (w_j^B)' &\leftarrow w_j^B; & (w_i^N)' &\leftarrow w_i^N;
 \end{aligned}$$

for  $i \in R \setminus \{I\}$  and  $j \in C \setminus \{J\}$

This completes the pivoting process.

#### 4. SIMPLEX PIVOT SELECTION SCHEMES:

There are four (two pairs) fundamental types of pivot selection schemes namely Primal Standard Pivot (PSP), Dual Standard Pivot (DSP), Primal Tricky Pivot (PTP) and Dual Tricky Pivot (DTP). Further a primal/dual standard pivot can again be classified into one with positive/negative indicator (that is PSPPI and DSPNI) and one with zero indicator (that is PSPZI and DSPZI), thus resulting in a set of possible six distinct types of pivot selection schemes, that are available for simplex pivoting process in solving linear programming problems. The algebra of the pivot selection schemes are given below, along with a schematic representation of the tableau data entries pattern that leads to such pivot selection.

#### 4.1 Dual Standard Pivot with Negative Indicator, DSPNI:

$$\begin{array}{cc|c}
 \bullet & - & - \\
 \bullet & \bullet & \oplus \\
 + & \theta & \bullet
 \end{array} \leftarrow \begin{aligned} & I \in \{ i \in R \mid \beta_i < 0 \}; \\ & J \leftarrow \arg \min_{j \in C} \{ (\gamma_j / \alpha_{ij}) \mid \beta_i < 0; \gamma_j \leq 0; \alpha_{ij} < 0 \} \end{aligned}$$

#### 4.2 Primal Standard Pivot with Positive Indicator, PSPPI:

$$\begin{array}{cc|c}
 \bullet & \bullet & - \\
 + & \bullet & \oplus \\
 + & \theta & \bullet
 \end{array} \rightarrow \begin{aligned} & J \in \{ j \in C \mid \gamma_j > 0 \}; \\ & I \leftarrow \arg \min_{i \in R} \{ (\beta_i / \alpha_{ij}) \mid \beta_i \geq 0; \gamma_j > 0; \alpha_{ij} > 0 \} \end{aligned}$$

#### 4.3 Primal Tricky Pivot (with Positive Indicator), PTPPI:

$$\begin{array}{cc|c}
 - & \bullet & - \\
 \bullet & \bullet & \oplus \\
 + & \theta & \bullet
 \end{array} \rightarrow \begin{aligned} & J \in \{ j \in C \mid \gamma_j > 0 \}; \\ & I \leftarrow \arg \max_{i \in R} \{ (\beta_i / \alpha_{ij}) \mid \beta_i < 0; \gamma_j > 0; \alpha_{ij} < 0 \} \end{aligned}$$

#### 4.4 Dual Tricky Pivot (with Negative Indicator), DTPNI:

$$\begin{array}{cc|c}
 + & \bullet & - \\
 \bullet & \bullet & \oplus \\
 + & \theta & \bullet
 \end{array} \leftarrow \begin{aligned} & I \in \{ i \in R \mid \beta_i < 0 \}; \\ & J \leftarrow \arg \max_{j \in C} \{ (\gamma_j / \alpha_{ij}) \mid \beta_i < 0; \gamma_j > 0; \alpha_{ij} > 0 \} \end{aligned}$$

#### 4.5 Dual Standard Pivot with Zero Indicator, DSPZI:

$$\begin{array}{cc|c}
 \bullet & \bullet & - \\
 \bullet & - & 0 \\
 \bullet & \bullet & + \\
 + & \theta & \bullet
 \end{array} \leftarrow \begin{aligned} & I \in \{ i \in R \mid \beta_i = 0 \}; \\ & J \leftarrow \arg \min_{j \in C} \{ (\gamma_j / \alpha_{ij}) \mid \beta_i = 0; \gamma_j \leq 0; \alpha_{ij} < 0 \} \end{aligned}$$

#### 4.6 Primal Standard Pivot with Zero Indicator, PSPZI:

$$\begin{array}{cc|c}
 \bullet & \bullet & - \\
 \bullet & + & \oplus \\
 + & 0 & - \\
 + & \theta & \bullet
 \end{array} \rightarrow \begin{aligned} & J \in \{ j \in C \mid \gamma_j = 0 \}; \\ & I \leftarrow \arg \min_{i \in R} \{ (\beta_i / \alpha_{ij}) \mid \beta_i \geq 0; \gamma_j = 0; \alpha_{ij} > 0 \} \end{aligned}$$

It can be easily observed that a primal (dual) tricky pivot with zero indicator is essentially the same as dual (primal) standard pivot with negative (positive) indicator, wherein the resultant minimum-ratio comes out to be zero.

## 5. EFFECT OF PIVOTING OPERATION:

It is useful at this point to make a few observations regarding the effect of pivoting operation, in each of the above pivot selection schemes.

DSPNI brings about an immediate improvement in the primal feasibility (w.r.t. the pivot row, at least) without deterioration of dual feasibility. The extent of this improvement in primal feasibility can be measured by the corresponding resultant improvement (decrease) observed in the value of the dual objective function, given by  $|\beta_1\gamma_j/\alpha_{1j}|$ .

PSPPI brings about an immediate improvement in the dual feasibility (w.r.t. the pivot column, at least) without deterioration of primal feasibility. The extent of this improvement in dual feasibility can be measured by the corresponding resultant improvement (increase) observed in the value of the primal objective function, again given by  $|\beta_1\gamma_j/\alpha_{1j}|$ .

PTPPI brings about an immediate improvement in the primal feasibility (w.r.t. the pivot row, at least) without any concern to the dual feasibility. The extent of this improvement in primal feasibility can be measured by the corresponding resultant improvement (increase) in the value of the ~~dual~~ <sup>primal</sup> objective function, again given by  $|\beta_1\gamma_j/\alpha_{1j}|$ .

DTPNI brings about an immediate improvement in the dual feasibility (w.r.t. the pivot column, at least) without any concern to the primal feasibility. The extent of this improvement in dual feasibility can be measured by the corresponding resultant improvement (increase) in the value of the ~~primal~~ <sup>dual</sup> objective function, again given by  $|\beta_1\gamma_j/\alpha_{1j}|$ .

The other two types of pivot selections namely DSPZI and DSPZI are utilized to affect a change in the basis, in situations with multiple-solutions and/or degeneracy, and do not in any way affect the primal or dual feasibilities. However they certainly play an important role in the overall solution strategy.

From the above, one can observe that as a general, uniform and common basis for the local effectiveness measure of a particular pivoting operation (applicable for any and every iteration, for both primal and dual) we can use the absolute value of the change in the value of the objective function, given by  $\text{lem}(I,J) = \text{abs}(\beta_1\gamma_j/\alpha_{1j})$ . It is not specifically suggested here, but one can opt to choose a pivot to maximize this local effectiveness measure in every iteration either among the possible pivots of a particular type or even among of all the possible pivots of all the possible types. Even if done so, it cannot be guaranteed (as per worst case analysis) to minimize the overall number of iterations required for reaching an optimum solution. It requires further research work to thoroughly understand, analyze and incorporate the concept of "local effectiveness measure (lem) of a single simplex pivoting operation" to the fullest extent, in developing an efficient solution strategy. For now, let us come to the main algorithm itself.

## 6. SYMMETRIC PRIMAL-DUAL SIMPLEX ALGORITHM:

The six distinct types of pivot selection schemes are considered in the following order of preference:

{(DSPNI, PSPPI)}, {PTPPI, DTPNI}, {DSPZI, PSPZI}

At every iteration, an attempt is made to select a pivot element, by checking the possible pivot selections belonging to one of the above six types of pivot selection schemes in the preference order pre-specified.

That is, a pivoting operation is executed with a pivot element of the type-

- i. DSPNI if possible,
  - ii. PSPPI only if none of DSPNI is possible,
  - iii. PTPPI only if none of DSPNI nor PSPPI is possible,
  - iv. DTPNI only if none of DSPNI, PSPPI, PTPPI is possible,
- and when none of the above four is possible, go for
- v. DSPZI and/or
  - vi. PSPZI in order to explore the possible alternative basic solutions.

It is to be noted that depending upon the actual data entries in the tableau, a pivot selection of specific type which was not possible in an earlier iteration, can become possible in a later iteration, sometimes even in the very next following iteration. That is why it is a crucial part of the algorithm to check in each and every iteration, for each of the six types of possible pivot selection schemes in the pre-specified order.

## 7. EVOLUTION OF TABLEAU DATA ENTRIES PATTERN:

As a means to show the convergence of the algorithm to one of the possible six termination patterns (discussed later), an analysis of the evolution of the tableau data entries pattern is given here. Figure-1 gives the six different patterns applicable when each of the

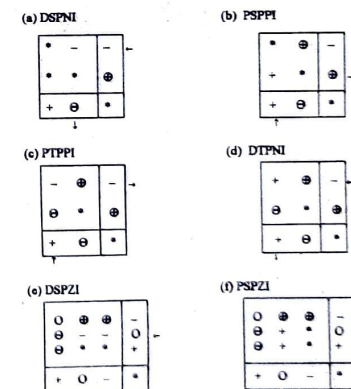


Figure 1. Evolution of tableau data entries pattern.  
- negative 0 Zero + positive  
x non-positive \* any value ⊕ non-negative





|       | D = F                                                                                                                                | D = ∞                                                                                                  | D = Φ |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
|-------|--------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|-------|---|---|---|---|---|--------------------------------------------------------------------------------------------------------------------------------------|---|--------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|---|---|---|---|---|---|---|---|--|
| P = F | <table><tr><td>*</td><td>⊖</td><td>0</td></tr><tr><td>⊕</td><td>*</td><td>+</td></tr><tr><td>0</td><td>-</td><td>●</td></tr></table> | *                                                                                                      | ⊖     | 0 | ⊕ | * | + | 0 | -                                                                                                                                    | ● | <table><tr><td>*</td><td>+</td><td>0</td></tr><tr><td>⊕</td><td>*</td><td>+</td></tr><tr><td>0</td><td>-</td><td>●</td></tr></table> | *                                                                                             | + | 0 | ⊕ | * | + | 0 | - | ● |  |
| *     | ⊖                                                                                                                                    | 0                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| ⊕     | *                                                                                                                                    | +                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| 0     | -                                                                                                                                    | ●                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| *     | +                                                                                                                                    | 0                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| ⊕     | *                                                                                                                                    | +                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| 0     | -                                                                                                                                    | ●                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| P = ∞ | <table><tr><td>*</td><td>⊖</td><td>0</td></tr><tr><td>-</td><td>*</td><td>+</td></tr><tr><td>0</td><td>-</td><td>●</td></tr></table> | *                                                                                                      | ⊖     | 0 | - | * | + | 0 | -                                                                                                                                    | ● |                                                                                                                                      | <table><tr><td>⊖</td><td>*</td><td>⊕</td></tr><tr><td>+</td><td>⊖</td><td>●</td></tr></table> | ⊖ | * | ⊕ | + | ⊖ | ● |   |   |  |
| *     | ⊖                                                                                                                                    | 0                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| -     | *                                                                                                                                    | +                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| 0     | -                                                                                                                                    | ●                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| ⊖     | *                                                                                                                                    | ⊕                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| +     | ⊖                                                                                                                                    | ●                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| P = Φ |                                                                                                                                      | <table><tr><td>⊕</td><td>-</td></tr><tr><td>*</td><td>⊕</td></tr><tr><td>⊖</td><td>●</td></tr></table> | ⊕     | - | * | ⊕ | ⊖ | ● | <table><tr><td>0</td><td>⊕</td><td>-</td></tr><tr><td>⊖</td><td>*</td><td>⊕</td></tr><tr><td>+</td><td>⊖</td><td>●</td></tr></table> | 0 | ⊕                                                                                                                                    | -                                                                                             | ⊖ | * | ⊕ | + | ⊖ | ● |   |   |  |
| ⊕     | -                                                                                                                                    |                                                                                                        |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| *     | ⊕                                                                                                                                    |                                                                                                        |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| ⊖     | ●                                                                                                                                    |                                                                                                        |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| 0     | ⊕                                                                                                                                    | -                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| ⊖     | *                                                                                                                                    | ⊕                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |
| +     | ⊖                                                                                                                                    | ●                                                                                                      |       |   |   |   |   |   |                                                                                                                                      |   |                                                                                                                                      |                                                                                               |   |   |   |   |   |   |   |   |  |

Figur 3. Six categories for tableau data entries Pattern at termination.  
 - negative 0 Zero + positive  
 ⊖ non-positive \* any value ⊕ non-negative • un-analyzed

Also it is possible to have alternative ordering of the pivot selection schemes incorporated in the algorithm within each of the three subsets indicated in section 6 above, although the relative ordering of these three subsets themselves is not to be altered.

Although rigorous mathematical analysis of the computational complexity of this algorithm is yet to be studied in detail, it seems very likely to perform at least as good as (if not better than) the existing solution techniques for linear programming.

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## GEOMETRIC GRAPHS

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### ABSTRACT

A  $(p, q)$ -graph  $G$  is said to be  $(a, r)$ -geometric if its vertices can be assigned distinct positive integers so that the values of the edges, obtained as the products of the numbers assigned to their end vertices, can be arranged in the geometric progression  $a, ar, ar^2, \dots, ar^{q-1}$ . In this paper we prove that some well known classes of graphs are geometric for certain values of  $a$  and  $r$  and also initiate a study on the structure of finite  $(a, r)$ -geometric graphs.

### 1. INTRODUCTION

For all standard terminology and notation in graph theory we refer Harary [3].

Several practical problems in real life situations have motivated the study of labelings of a graph  $G = (V, E)$ , which are required to obey certain variety of conditions depending upon the structure of  $G$  such as adjacency. There is an enormous amount of literature built up on several kinds of labelings of graphs over the past three decades or so, and it would be too unwieldy to list them here. For a cursory look at the topic we refer the reader to Bloom [2].

Labeling of  $G$  is merely an assignment of certain distinct nonnegative integers to the vertices of  $G$ .

In this paper we are interested in the study of vertex functions  $f: V(G) \rightarrow \mathbf{A}$ ,  $\mathbf{A} \subseteq \mathbf{N}$ , for which the induced edge function  $f^*: E(G) \rightarrow \mathbf{N}$  is defined as  $f^*(uv) = f(u)f(v)$ ,  $\forall uv \in E(G)$ . Such vertex functions are said to be **multiplicative** and henceforth this particular map  $f^*$  of  $f$  will be denoted  $f^\lambda$ .

**THEOREM 1** (Acharya and Hegde [1]): For any graph  $G$  and for any multiplicative vertex function  $f: V(G) \rightarrow \mathbf{N}$

$$(1) \quad \prod_{e \in E(G)} f^\lambda(e) = \prod_{u \in V(G)} f(u)^{d(u)}.$$

**COROLLARY 1.1**: If  $G$  is an  $r$ -regular graph then for any multiplicative vertex function  $f$  of  $G$ ,

$$\prod_{e \in E(G)} f^\lambda(e) = \prod_{u \in V(G)} f(u)^r.$$

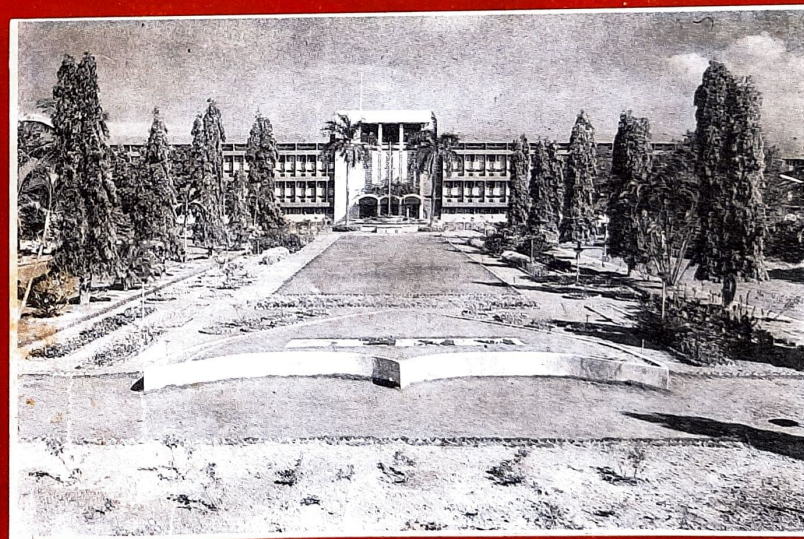
### 2. MULTIPLICATIVE NUMBERINGS

A **multiplicative numbering** of a graph  $G$  is an injective multiplicative vertex function  $f$  such that the induced edge function  $f^\lambda$  is also injective.



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